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Research note

Topology design of optimizing material arrangements of beam-to-column connection frames with maximal stiffness

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Abstract This study presents conceptually effective layouts of materials, i.e. steel or fiber, optimally positioned into building frames. The design information of the material layout may be helpful in dealing with large-scale safety design issues in civil or architectural engineering fields, against natural phenomena, such as winds and earthquakes. The material topology optimization method evaluates an optimal layout reinforcing or arranging material of a specified volume in a given design space that maximizes stiffness for a given set of loads and boundary conditions. Generating the optimal distribution of material is similar to the so-called strut-and-tie method using truss members of straight lines, and it leads to the stiffest structures. Numerical applications verify that the present material topology optimization method is an applicable concept design tool to create effective layout designs of material in given structural frames in civil and building industries.

1. Introduction

In the fields of civil and architectural engineering, large-scaled structures, such as bridges and buildings, are mainly treated for structural design. In large scale structural design, structural safety is most significant due to its importance in human safety. In addition, economical aspects, like construction cost, life cycle cost, and sustainability, have also to be considered when structural designs are carried out.

In general, both linear and non-linear strain distributions exist in mechanical objects, which respond to given loading and boundary conditions, especially in large-scaled structures. Stress is more concentrated near loading positions and jagged surfaces due to the intensity of linear and non-linear strain distributions, when structures have geometrically discontinuous surfaces.

The linear strain distribution areas defined by the Bernoulli theory are denoted as the *B* (Bernoulli) region [1]. The non-linear strain distribution occurs mainly in points or corners of applied loads and discontinuous surfaces. Stress concentrations, as shown in Figure 1b, appear at these points or at the corners of structures like beam-to-columns in Figure 1a, and deep beams. It is termed a *D* (Disturbance) region [1], as shown in areas represented by diagonal lines in Figure 2.

D regions are weak positions in which structural damage like cracks, as shown in Figure 1c, may occur. *D* regions have to be reinforced by inserting new material like steel or fiber, if necessary, to stop cracks and guarantee structural safety.

Conventionally, reinforcement about *D* regions is qualitatively designed by experience or by the decision of experts. A so-called strut-and-tie model design [1], introduced by Schlaich et al., is an alternative to the non-quantitative reinforcement design of *D* regions. The strut-and-tie model design has several advantages [2], and this method has recently been selected as the design criteria like FIP [3] in 1996 and ACI [4] in 2002 for the reinforcement of concrete structures.

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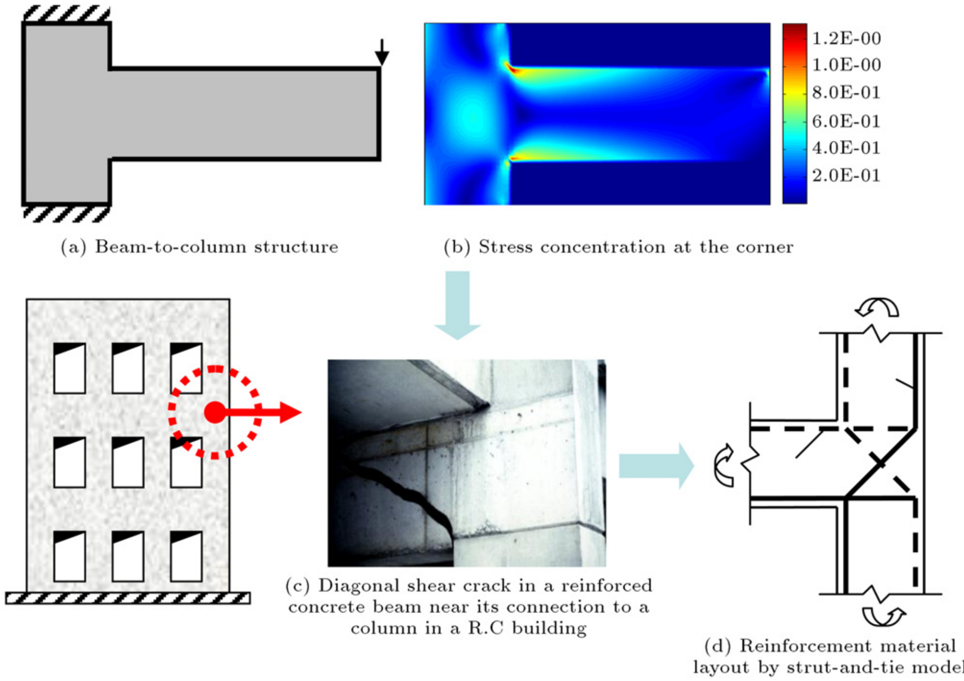


(a) Beam-to-column structure

(b) Stress concentration at the corner

(c) Diagonal shear crack in a reinforced concrete beam near its connection to a column in a R.C building

(d) Reinforcement material layout by strut-and-tie model

Figure 1: Stress concentration and its reinforcement.

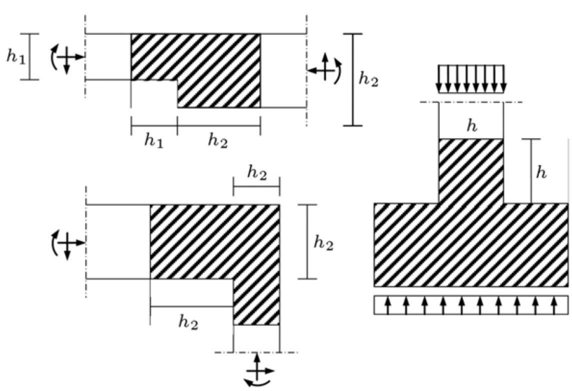


Figure 2: *D*-region: beam-to-columns and foundation.

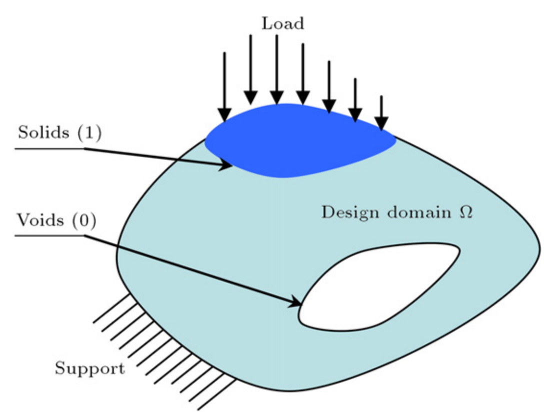


Figure 3: Design space for two-phase material topology optimization problems of structures.

However, the conventional strut-and-tie model designs require a trial-and-error procedure in order to achieve the reinforcement design about a given structure. Although the strut-and-tie model design is conceptually simple, a straight unit, like a truss of the strut-and-tie model, has limits wherein it is not possible to geometrically change the inside of an assigned straight truss in *D* regions. *Moreover, the arranged direction and thickness of struts and ties are determined by compression or tension of principal stress lines, calculated using linear truss finite element analyses.*

In order to quantitatively and consistently carry out the reinforcement design, a scientific, automatic tool is needed, not the conventional strut-and-tie model design created by human trial and error [5,6]. The idea of a material topology optimization [7,8] tool, presented in this study, determines the optimal layout of material of a specified volume in a given design space that maximizes stiffness for a given set of loads and boundary conditions. SIMP (Solid Isotropic Microstructure of Penalization for Intermediate Density) [7,8], which is well-known in material topology optimization methods, is utilized in this study.

The optimal assignment or layout of material for reinforcement is similar to the strut-and tie model, and it leads to the stiffest building structure. Both static and dynamic problems are considered in this study in order to try the generalized reinforcement design tool, including total structural behavior. For a static problem, the objective is minimal strain energy. For a dynamic topology optimization problem, the objective is related to maximizing the first-order eigen-frequency, subject to a given material limit, since structures with a high fundamental frequency tend to be reasonably stiff for static loads.

The outline of this study is as follows: The material topology optimization formulations for static and free vibration problems are described in Section 2 including SIMP material formulation. Section 3 shows a numerical algorithm of material topology optimization for static and dynamic problems. Numerical applications that verify that a typical material topology

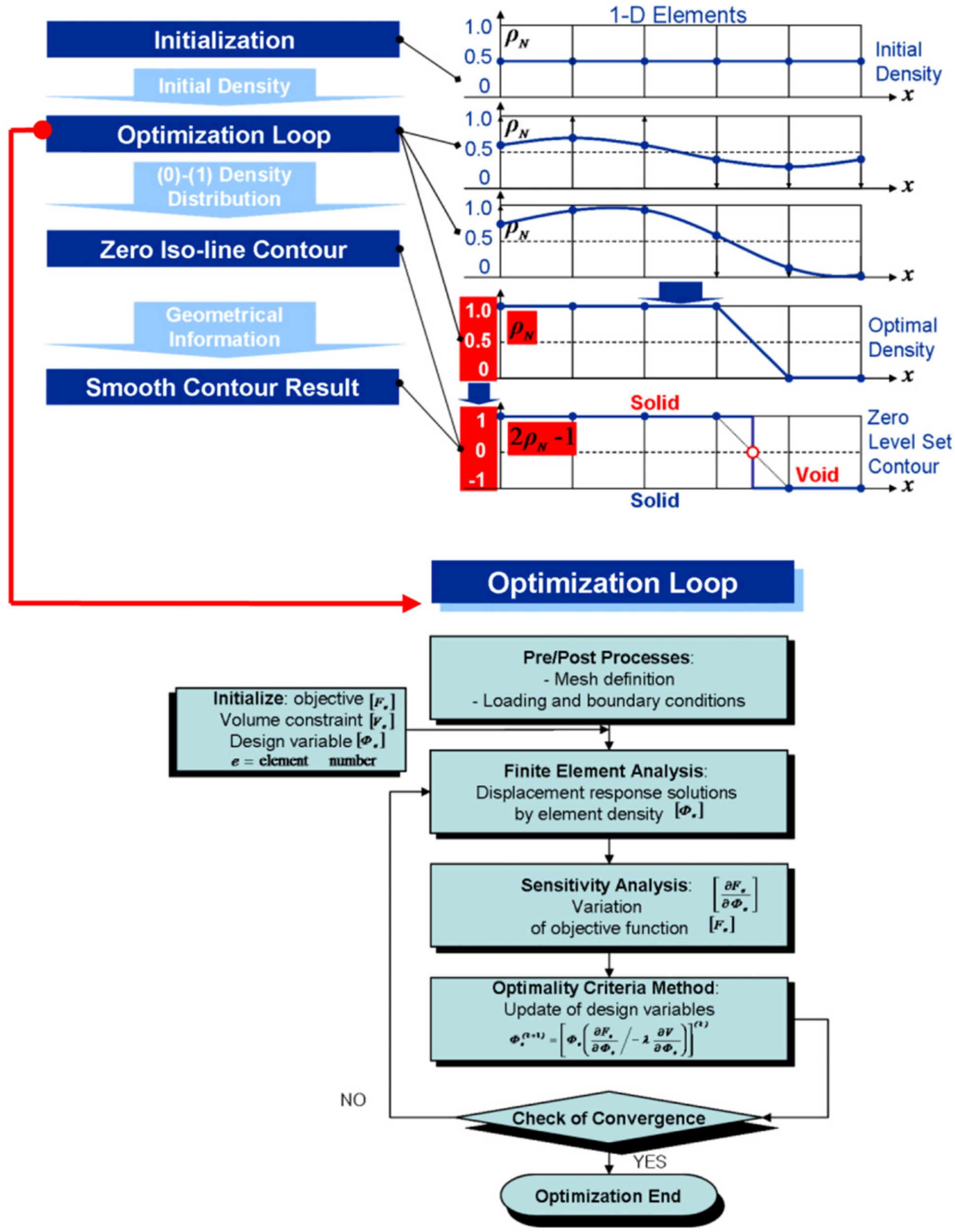


Figure 4: Numerical algorithm of material topology optimization.

optimization method is an applicable design tool for optimizing layouts of reinforced materials into a given building structure are presented in Section 4. Finally, Section 5 presents the conclusions of this study.

2. Topology optimization formulations for static and dynamic problems

2.1. Optimization formulations for static problems

The general problem of structural topology optimization is specified as the objective function and constraints. Please note that according to the principle of minimum potential energy, the objective function can be written as minimum compliance, i.e. minimal strain energy for static problems, as follows. The minimal compliance problem aims to design the stiffest or least compliant structure using a given fixed load, possible support conditions, and restrictions on the volume of material used in a given design space, as shown in Figure 3:

$$f = \frac{1}{2} \int_{\Omega_x} \delta \boldsymbol{\varepsilon}^{\mathrm{T}} \mathbf{C} \varepsilon \, d\Omega_x, \tag{1}$$

where according to discretization, the continuous material tensor, $\mathbf{C}$, is dependent on the density-stiffness relationship of the typical SIMP approach. The discontinuous Heaviside function is regularized for a smoothed and continuous form near the material boundaries. The function can be included in a strain energy formulation, since the original Heaviside function determines the solid and void regions in a design domain.

The inequality optimization constraint is $0 \le \Phi \le 1$, which ensures that the density stays within reasonable bounds. Equality constraints are linear elastostatic equilibrium, which

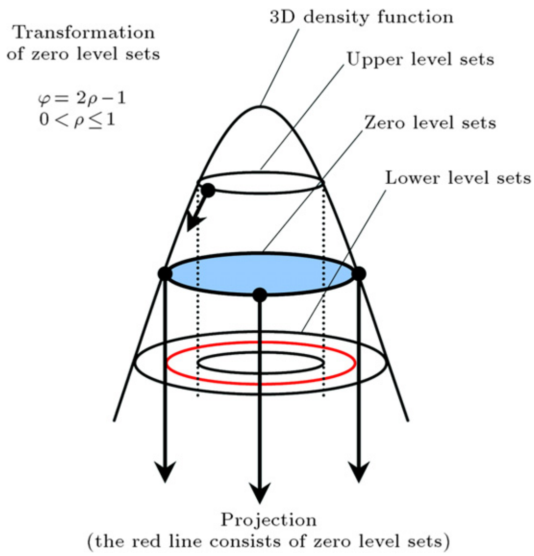


Figure 5: Zero level set function, φ.

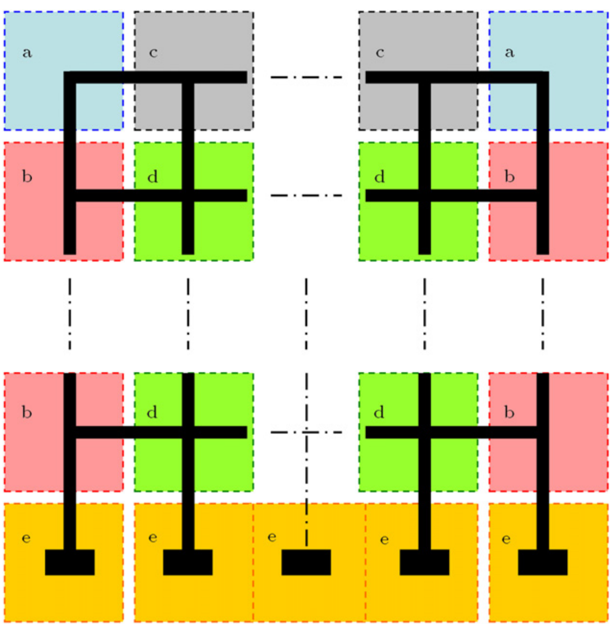


Figure 6: *D*-region in a typical building structure (beam-to-columns, foundations).

clearly presents the state equation, and an equation controlling the volume of the used material under the volume fraction, V_{ref}, respectively, as follows:

$$\int_{\Omega_x} \delta \boldsymbol{\varepsilon}^T \mathbf{C} \varepsilon \, d\Omega_x = \int_{\Omega_x} \delta \mathbf{u}^T \bar{\mathbf{b}} \, d\Omega_x + \int_{\Gamma_t} \delta \mathbf{u}^T \bar{\mathbf{t}} \, d\Gamma_t, \tag{2}$$

$$\int_{\Omega_x} d\Omega_x - V_{ref} = 0. \tag{3}$$

2.2. Optimization formulations for dynamic structural free vibration problem

Eigenvalue optimization designs are profitable for mechanical structural systems subjected to dynamic loading conditions, like earthquakes and wind loads. The dynamic behavior of structural systems can be estimated by eigen-frequency, which describes structural stiffness. In general, maximizing the first-order eigen-frequency can be an objective for dynamic topology optimization problems [9,10], since the stiffness of structures also increases when eigenfrequency increases. Problems of topology optimization for maximizing the natural eigen-frequencies of vibrating elasto-static structures have been considered in studies [9,10].

Assuming that damping can be neglected, such a dynamic design problem can be formulated as follows. Here, neglecting damping is to consider, substantially, minimum and maximum problems of structures without additional mechanical implementation:

$$\max_{\underline{\Phi}} : \omega_1^2 (\Phi) = \frac{\mathbf{u}_1^T \mathbf{K} \mathbf{u}_1}{\mathbf{u}_1^T \mathbf{M} \mathbf{u}_1}, \tag{4}$$

$$\text{Subject to: } \frac{V(\Phi)}{V_0} \leq g, \tag{5}$$

$$: \left[\mathbf{K} - \omega_i^2 \mathbf{M}\right] \mathbf{u}_i = \underline{0}, \tag{6}$$

$$: 0 < \Phi_{\min} \leq \Phi \leq \Phi_{\max}, \tag{7}$$

where ω_1 denotes the first-order eigen-frequency, i.e. a given objective function, depending on design variable Φ, and $\mathbf{K}$ and $\mathbf{M}$ are global stiffness and mass matrices, respectively. Both matrices depend on the penalization of design variable, Φ, as shown in Section 2.3. A consistent, and a lumped mass, and a combination of both, such as in the present study, can be used for $\mathbf{M}$. The inequality optimization constraint is $0 < \Phi_{\min} \leq \Phi \leq \Phi_{\max}$ of Eq. (7). In order to escape numerical singularity, the limit of Φ is given as $\Phi_{\min} = 0.001$ and $\Phi_{\max} = 1.0$. Equality constraints are provided by the dynamic free vibration equation of (6), and the limit on the required amount, $V(\Phi)$, of material in terms of the constant volume V_0 of the design domain of Eq. (5). g is the ratio between an obtained volume, $V(\Phi)$, and a given volume constraint, V_0.

2.3. General principles of SIMP method [7,8]

The goal of topology optimization is to provide the optimal material distribution into a restricted space, i.e. the design space. For this purpose, the principle is to cut the design space into small finite elements and to determine which ones belong to the solution.

Optimization variables correspond to the densities of each finite element. The relative density may take any value between 0 and 1, and an artificial material law (SIMP—Simply Isotropic Material with Penalization) is implemented to link together stiffness and density, as follows:

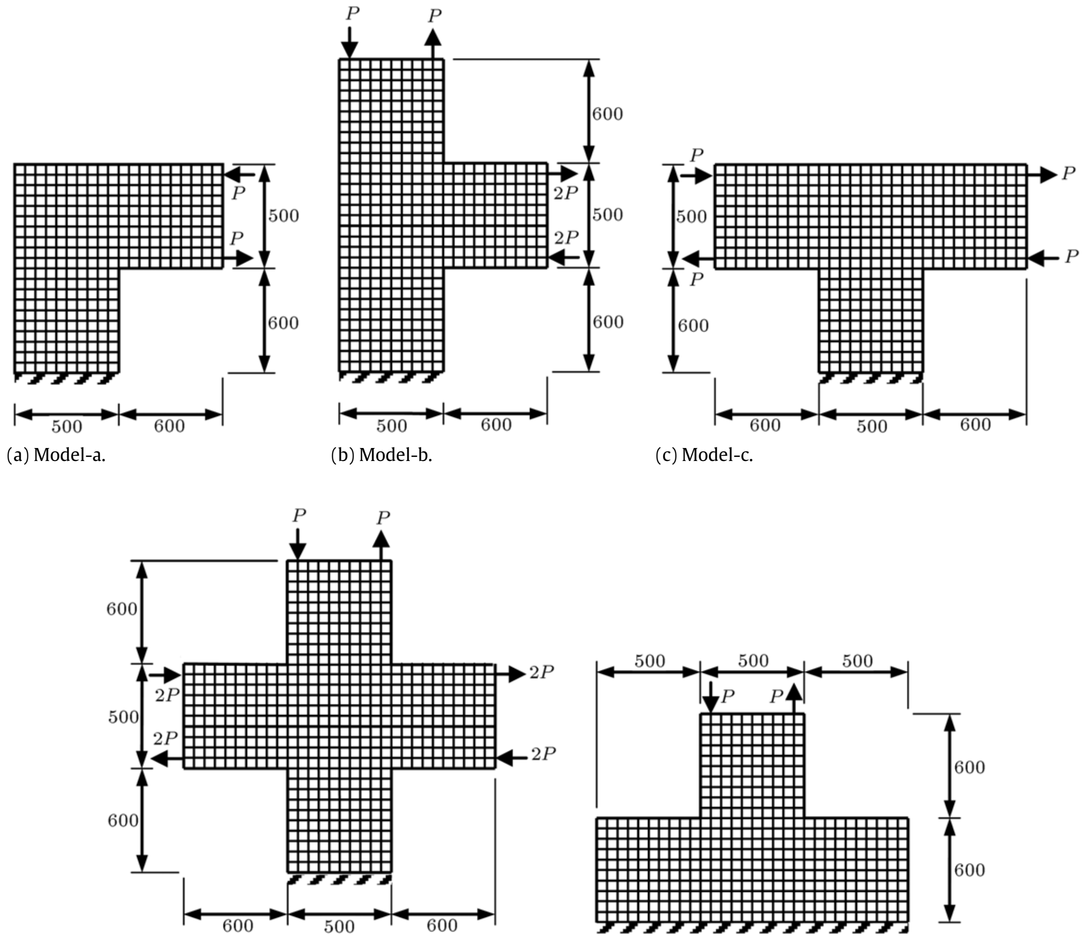
$$E_i^h \left(\Phi_i^h\right) = E_0 \left(\frac{\Phi_i^h}{\Phi_0}\right)^k, \quad k \geq 1, \ 0 \leq \Phi_i^h \leq 1,$$
$$i = 1 \cdots N_e, \tag{8}$$

where E_0 and Φ_0 denote nominal values of Young's modulus and the material density of elements, respectively, and N_e is the number of elements. k is the penalization factor and Φ_i is the relative density of element i.

In this study, these penalization formulations are used for dynamic (such as in Section 2.2), as well as static (such as in Section 2.1) optimal designs in this study.

3. Numerical algorithm of material topology optimization

The SIMP material topology optimization algorithm has numerical steps of the following subsections, as shown in



(a) Model-a. (b) Model-b. (c) Model-c.

(d) Model-d. (e) Model-e.

Figure 7: Structural Models a–e for which material reinforcements are needed (an example of Liang et al. [15]).

Figure 4. Here, a MATLAB code introduced by Sigmund [8] is an SIMP program for static problems, and the program was extended for dynamic free vibration topology optimization.

3.1. Initialization

Relative element densities are assumed as design parameters of topology optimization. For the optimization model, the objective function and constraints are defined. The geometry, boundary, and loading conditions of a structure are defined in a given design space. By considering an assumed material volume fraction, initial design parameters are constantly assigned into the design space.

As can be seen in Figure 4, Φ_i of Section 2.3 is assumed ρ_N in one dimensional space (1-D), and then a material volume fraction of 50% is assigned into the design space.

3.2. Optimization loop

Element densities move toward voids (almost 0 due to numerical singularity) and solid regions (1) during topology optimization steps as illustrated in the algorithm in Figure 4. After some iterations, they converge to (0)–(1) density distributions in the design space. Densities are used as material properties of each element, when FEM is performed. SIMP material is used for this optimization step.

Topology optimization steps consist of three stages [11]: a structural analysis, such as the finite element method, a sensitivity analysis [12], and an optimization, such as the optimality criteria [13] method.

3.3. Zero level set contour process

In a typical situation for two-phase material topology optimization problems, the interface between solid and void phases can be represented by a specific level set function [14]. Figure 5 shows that zero level sets are extracted from an element density function. By $\varphi \leftarrow 2\rho - 1 \ (0 \prec \rho \leq 1)$, the level set function, φ, is produced, and, finally, the middle position of $-1 \leq \varphi \leq 1$, i.e. 0, are zero level sets. The use of the level set function provides numerical efficiency due to the boundary description associated with a fixed regular mesh. This is used in order to achieve smooth shapes for topologies in this study.

4. Numerical applications and discussion

Numerical examples involve generating proper layouts of materials, here steel inserted for reinforcement of a given building structure, using the design tool of a continuous two-phase (0–1) material SIMP topology optimization in cases of linear elastostatic and free vibration problems. The term

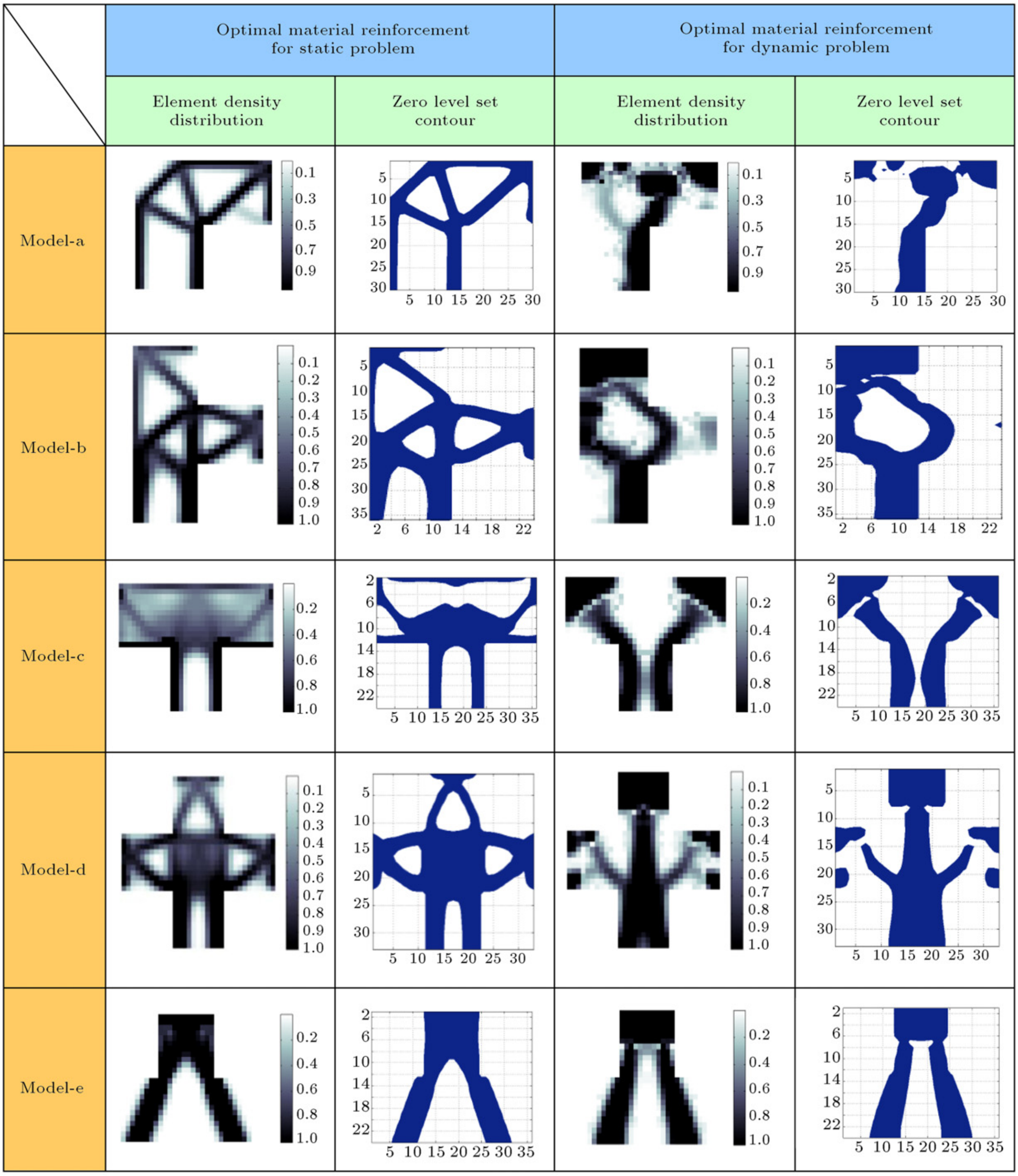


Figure 8: Optimal material reinforcement layouts for static and dynamic problems.

"proper" means "both the most economical and the stiffest". A given design space is discretized by square bilinear elements. The material parameters, here concrete for a given building structure, are assumed as Young's modulus, $E = 20$ MPa, and material for reinforcement is $E = 200$ MPa. Both cases are Poisson ratio, $\nu = 0.25$. A plane stress state is assumed for the design space. The objective function is the minimal strain energy (kg cm) for static problems, and the maximal first-order eigenfrequency for dynamic problems. The material volume fraction is limited to a given value during every optimization procedure. The optimization problem is solved by optimality criteria. In order to obtain numerical stability, filter methods are used and the filter radius is 2.0. The penalty parameter of SIMP material is 3.0.

4.1. Concrete building frame structure

In general, frame structures are a bending, moment-resistant, structural system in which beams are connected to columns. *D*-regions (beam-to-columns; a–d) and foundations (e) in the concrete frame structure, as shown in Figure 6, are used for structural models.

Model a is an opening knee joint at the right and left summit of the building. Model b is an exterior beam-to-column, in

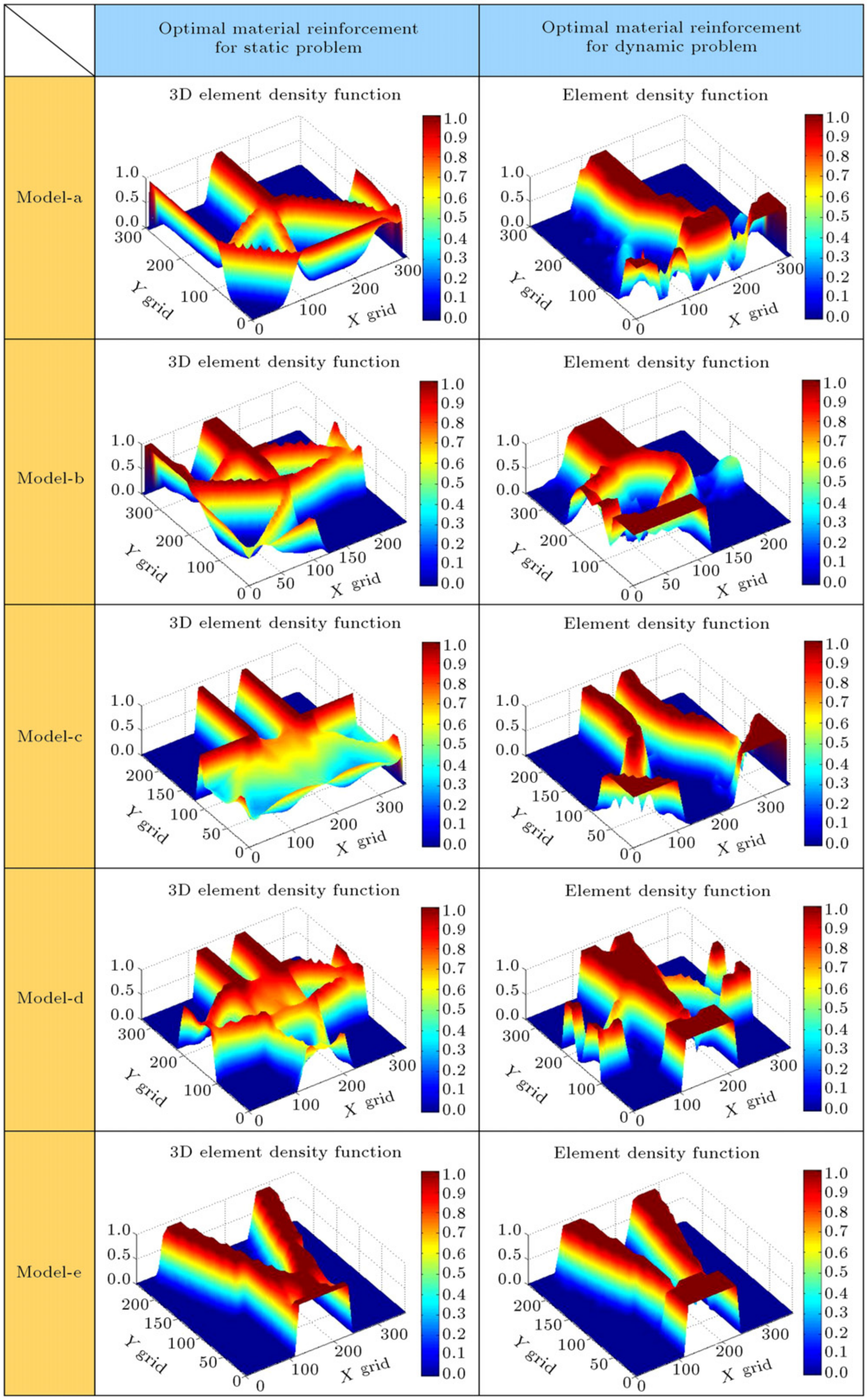


Figure 9: Three-dimensional density distributions of optimal layouts for static and dynamic problems.

which a beam meets an exterior continuous column. Model c is also an exterior beam-to-column in which one column is connected to an exterior continuous beam. Model d is cross-shaped, beam-to-column, and Model e is a foundation resistant to bending moments.

Input data for the structural and analysis properties for optimization are shown in Table 1. Geometry, loading, and boundary conditions of Models a–e are shown in Figure 7.

Figure 8 shows optimal material layouts, reinforced by steel material of 30% or 35% of total volume, in cases of static and

Table 1: Design input data.

Structural properties			Analysis properties		
Geometric discontinuity (*D*-region)		Material *E*-module (MPa)	Finite elements	Volume constraint	Load *P* (kN)
Beam-to-columns	Model-a	Conc.: 20 steel: 200 (Poisson's ratio = 0.25)	30 × 30	30	500
	Model-b		24 × 36	35	
	Model-c		36 × 24	35	
	Model-d		33 × 33	35	
Foundations	Model-e		36 × 24	30	

dynamic problems. The results indicate that optimal layouts are necessary in order to make the stiffest concrete building frame, if additional material (here, steel) has to be inserted for reinforcement. Structural Models, a–e, resist bending moments. Then, the optimal topology layout produces computational representations, optimally resistant to complex currents of interior forces, which the structural model absorbs using bending moments. *In* Figure 8, *strange shapes in dynamic results are owing to numerical singularities, such as stress concentrations at corners, using non-symmetric initial design space*.

The layouts can be presented by constant density distributions of 0 and 1 into each element, and a zero level set contour of an element density function, which is shown in Figure 9. The zero level set contour produces smooth optimal results, not jagged boundaries, such as element density distributions. They are produced by topology optimization procedures, as shown in Section 3. As can be seen, these layouts are similar to strut-and-tie models that consist of truss members. The best material layouts would be a combination of the two results, to cover both static and dynamic problems.

Figure 9 presents three dimensional element density functions as optimal results, after optimization procedures in static and dynamic problems. A zero level set function, as shown in Figure 8, can be found by the continuous function.

Note that the present topology optimization tool produces optimal depositions of reinforcement materials. Here, the quantity of reinforced materials, for example, 30% of the total volume, can be selected by design conditions, and the results are automatically generated. It can be found that the easy compatibility has the highest merit in the present optimization tool.

5. Conclusions

This study presents the generation of automatic material depositions in material reinforcement, using the classic, element-based, SIMP material, topology optimization method, for both static and dynamic problems.

Generating the optimal distribution of material is similar to the so-called strut-and-tie method, using truss members of straight lines, which leads to the stiffest structures. Clearly, the coupling of the strut-and-tie model design and SIMP topology optimization has proven to be a very promising synthetic·-automatic strut-and-tie model design. The superiority of the present method enables it to be more practical and physically applied to three-dimensional and nonlinear problems.

Acknowledgment

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